

Collisional Hardening of Compact Binaries in Globular Clusters

S. Banerjee¹ and P. Ghosh¹

¹*Tata Institute of Fundamental Research, Mumbai, India.*

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ABSTRACT

We consider essential mechanisms for orbit-shrinkage or “hardening” of compact binaries in globular clusters to the point of Roche-lobe contact and X-ray emission phase, focussing on the process of collisional hardening due to encounters between binaries and single stars in the cluster core. The interplay between this kind of hardening and that due to emission of gravitational radiation produces a characteristic scaling of the orbit-shrinkage time with the single-star binary encounter rate γ in the cluster which we introduce, clarify, and explore. We investigate possible effects of this scaling on populations of X-ray binaries in globular clusters within the framework of a simple “toy” scheme for describing the evolution of pre-X-ray binaries in globular clusters. We find the expected qualitative trends sufficiently supported by data on X-ray binaries in galactic globular clusters to encourage us toward a more quantitative study.

Key words: globular clusters: general – scattering – stellar dynamics – binaries: close – stars: low-mass – stars: horizontal branch – X-rays: binaries

1 INTRODUCTION.

It is well-known that globular clusters contain far more than their fair share of compact X-ray binaries per unit stellar mass, compared to their host galaxies (Verbunt & Hut 1987, Verbunt & Lewin 2004). The enhancement factor is ~ 100 in the Milky Way and M31 (Verbunt & Lewin 2004, Pooley et.al. 2003), and possibly much higher in elliptical galaxies, as recent *Chandra* observations have suggested (Angelini et.al. 2001, Pooley et.al. 2003). The origin of this overabundance of close binaries has been realized for some thirty years now to be the dynamical formation of such binaries — through tidal capture and/or exchange interactions — which can proceed at a very significant rate in dense cores of globular clusters (henceforth GCs) because of the high stellar-encounter rates there (Hut 1985, Hut & Verbunt 1983, Hut et.al. 1992), but whose rate is negligible over the rest of the galaxy, where the stellar density is low by comparison. The GC X-ray binaries that we shall be mainly concerned with in this work are those which are powered by accretion onto compact stars. These can be either (a) low-mass X-ray binaries (henceforth LMXBs), containing neutron stars accreting from low-mass companions, or, (b) cataclysmic variables (henceforth CVs), containing white dwarfs accreting from low-mass companions. Accordingly, we shall not explicitly consider here binaries which contain either (a) two “normal” solar-mass stars, one or both of which are coronally active, or, (b) recycled neutron stars operating as rotation-powered millisecond pulsars, with a white-dwarf or

a low-mass normal companion, although such binaries can be low-luminosity X-ray sources. However, general considerations on the dynamical formation of close binaries do apply to these as well; indeed, the latter binaries are now widely accepted as evolutionary products of LMXBs (van den Heuvel 1991, 1992).

In tidal-capture formation of a compact-star binary, the compact star (neutron star/white dwarf) passing close to a normal star dissipates its kinetic energy significantly by creating tidal deformation in the latter star, and so becomes bound to it. In the exchange process of formation, a compact star replaces one of the stars of an existing binary system of two normal stars during a dynamical encounter (Hut 1985, Hut & Verbunt 1983, Spitzer 1987). After formation in such an encounter, the compact-star binary continues to undergo stellar encounters in the dense cores of GCs, and it is on one particular effect of the continuing encounters that we focus in this paper. In the mid-1970s, it was realized that a major effect of the binary-single star encounters would be to extract energy from a given binary, making it more tightly bound or *harder*, and giving this energy to the motion of the single stars in the GC, thus “heating” the cluster (Heggie 1975, Spitzer 1987, Hut et.al. 1992). We can call this effect *collisional hardening* of the compact-star binary, which makes the binary’s orbit shrink at a rate higher than that which would obtain if it were not subject to the above stellar encounters, *i.e.*, if it were not in a GC. The latter rate is believed to be determined by a combination of two processes, *viz.*, (a) emission of gravitational radiation and (b)

magnetic braking. In the former process, a compact binary emits gravitational radiation and so loses energy and angular momentum, which makes its orbit shrink (d’Inverno 1992). The latter mechanism is envisaged as follows. The low-mass companion to the compact star has a significant magnetic field, and also has its rotation tidally coupled or “locked” to that of orbital revolution. The companion drives a wind, which carries away angular momentum at an enhanced rate because the magnetic field enforces co-rotation of the wind out to a radius considerably larger than that of the star, and this angular momentum ultimately comes from the orbit because of the above tidal locking, thus making the orbit shrink (Verbunt & Zwaan 1981). We discuss in this paper the relative roles of the above mechanisms for binary hardening, particularly the role of collisional hardening *vis-à-vis* that due to gravitational radiation, indicate and clarify a scaling that naturally emerges from this interplay, and briefly suggest possible observational signatures of this scaling.

In Sec. 2, we discuss the hardening of compact binaries by the three mechanisms discussed above, bringing out the particular role of collisional hardening. We show that a characteristic scaling of the orbit-shrinkage time of such binaries with an essential GC parameter emerges because of the interplay between collisional hardening and that due to gravitational radiation. In Sec. 3, we explore possible observational signatures of this scaling. We sketch a very simple “toy” scheme for describing the evolution of compact X-ray binaries in GCs, and indicate a possible signature of the above scaling within the bounds of this toy model. We show that current data on X-ray binaries in GCs are consistent with this signature. We discuss our results in Sec. 4, exploring possible lines of future inquiry.

2 HARDENING OF COMPACT BINARIES.

We consider the shrinking of the orbital radius of a compact binary by the three mechanisms introduced above. Consider gravitational radiation first. The rate at which the radius a of a binary decreases due to this process is given by (see, e.g., d’Inverno 1992 and references therein):

$$\dot{a}_{\text{GW}} \equiv \alpha_{\text{GW}} a^{-3}, \quad \alpha_{\text{GW}} \approx -12.2 M m_X m_c R_\odot / \text{Gyr} \quad (1)$$

In this equation, m_X and m_c are respectively the masses of the compact star and its companion in solar masses, $M \equiv m_X + m_c$ is the total mass in the same units, and the orbital radius a is expressed in units of solar radius. We shall use these units throughout this paper.

Now consider magnetic braking. The orbit shrinkage rate due to this process is given in the original Verbunt-Zwaan prescription (Verbunt & Zwaan 1981) as:

$$\dot{a}_{\text{MB}} \equiv \alpha_{\text{MB}} a^{-4}, \quad \alpha_{\text{MB}} \approx -190 \frac{M^2}{m_X} \left(\frac{R_c}{a} \right)^4 R_\odot / \text{Gyr} \quad (2)$$

where R_c is the radius of the companion. We give below further discussion on this mechanism.

Finally consider the rate of orbit shrinkage due to collisional hardening, which is given by (Shull 1979),

$$\dot{a}_{\text{C}} \equiv \alpha_{\text{C}} \gamma a^2, \quad \alpha_{\text{C}} \approx -2.36 \times 10^{-7} \frac{m_{\text{GC}}^3}{m_X m_c} R_\odot / \text{Gyr} \quad (3)$$

Here, m_{GC} is the mass of the single stars in the GC which are undergoing encounters with the binary. In this introduc-

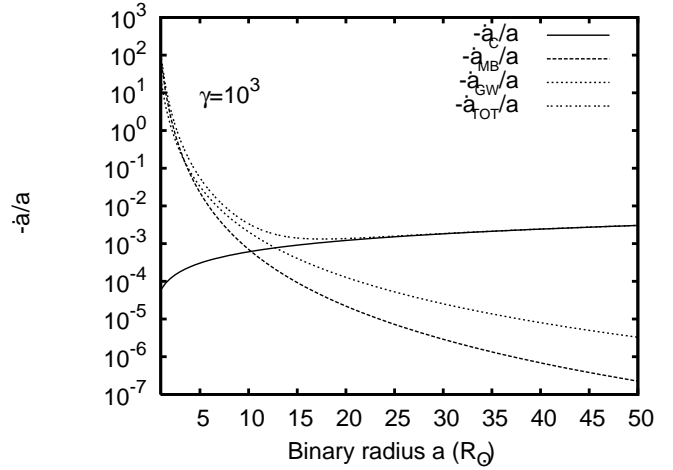


Figure 1. Relative orbit shrinkage rates $-\dot{a}/a$ due to gravitational radiation, magnetic braking and collisional hardening, shown as functions of the binary separation a . Also shown is the total shrinkage rate. Value of γ as indicated.

tory work, we assume m_{GC} to be a constant, representing a suitable average value for a GC core, which we take to be $m_{\text{GC}} \approx 0.6 M_\odot$.

The parameter γ is a measure of the encounter rate between a given binary and the background of single stars in the core of the GC: it is a crucial property of the GC for our purposes, so that we shall use it constantly in this work. It scales as $\gamma \propto \rho_c / v_c$ with the (average) core density ρ_c of the GC, and the velocity dispersion v_c of the stars in the core. Following the convention often employed in the GC literature (Verbunt 2002, Hut 1985), we can, in fact, define this parameter as:

$$\gamma \equiv \frac{\rho_c}{v_c}. \quad (4)$$

Then the unit of γ is $\approx 6.96 \times 10^5 M_\odot R_\odot^{-4} \text{ sec}$, corresponding to the units of ρ_c and v_c commonly used in the GC literature, namely, $M_\odot \text{ pc}^{-3}$ and km sec^{-1} respectively. In these units, values of γ generally run in the range $\sim 10^3 - 10^6$ (see below).

In this work, we take the mass of the compact star to be $m_X = 1.4 M_\odot$ and that of the companion $m_c = 0.6 M_\odot$, the latter corresponding to a typical average mass of stars in a GC (see above). According to the mass-radius relation for low-mass main-sequence stars, the radius of a main-sequence companion will then be $R_c \approx 0.6 R_\odot$. Furthermore, we consider only circular orbits in this work, returning to this point in Sec. 4.

The total rate of orbit shrinkage due to the combination of the above mechanisms is given by:

$$\dot{a} = \dot{a}_{\text{GW}} + \dot{a}_{\text{MB}} + \dot{a}_{\text{C}} \quad (5)$$

We emphasize that the first two terms in Eq. (5) are always operational, irrespective of whether the binary is in a GC or not, and it is the relative effect of the third term, which represents the effects of the encounters in a GC core, that we wish to study here. The interplay between the first and the third term was investigated in a pioneering study by Shull (1979), before the magnetic braking mechanism was postulated (Verbunt & Zwaan 1981).

Note first that the three terms have different regions of dominance, as shown in Fig. 1. Collisional hardening dominates at large values of the orbital separation a , *i.e.*, for wide binaries, while hardening by gravitational radiation and magnetic braking dominates at small a , *i.e.*, for narrow binaries. Between the latter two, magnetic braking dominates at the smallest orbital separations, if we adopt the original Verbunt-Zwaan (henceforth VZ) scaling for it (see below). The relative orbit shrinkage rate \dot{a}/a thus scales as a at large orbit separations, passes through a minimum at a critical separation a_c where the gravitational radiation shrinkage rate, scaling as $\dot{a}/a \sim a^{-4}$, takes over from collisional hardening, and finally rises at very small separations as $\dot{a}/a \sim a^{-5}$ due to VZ magnetic braking. The change-over from gravitational radiation shrinkage to that due to magnetic braking occurs at a radius $a_m \ll a_c$. These two critical radii are easily obtained from Eqs. (1), (2), and (3), and are given by

$$a_c = \alpha_{\text{GW}}^{1/5} \alpha_{\text{C}}^{-1/5} \gamma^{-1/5}, \quad a_m = \frac{\alpha_{\text{MB}}}{\alpha_{\text{GW}}} \quad (6)$$

Note the scaling $a_c \propto \gamma^{-1/5}$, which is crucial for much of our discussion here, as we shall see below. The critical orbital separation a_c varies in the range $\sim (5 - 12)R_\odot$ for the canonical range of values of the above GC parameter (Verbunt 2002) $\gamma \sim 10^3 - 10^5$ in the above units (Shull 1979).

The relevance of this to close compact-star binaries in GCs is as follows. When such a binary is formed, its orbital separation in most cases is such that the low-mass companion is not in Roche lobe contact, since the Roche-lobe radius has to be $R_L \sim 0.6R_\odot$ or less for this to happen for a typical low-mass main sequence or subgiant companion of mass $\sim 0.6M_\odot$ (see above). Mass transfer does not occur under such circumstances, so that such binaries are pre-LMXBs or pre-CVs, and we can call them by the general name *pre-X-ray binaries*, or PXBs for short. It is the above orbit-shrinkage or hardening process that brings the companion into Roche-lobe contact, so that mass transfer begins, and the PXB turns on as an X-ray binary (LMXB or CV), or XB for short.

Depending on the initial separation a_i of the binary, some or all of the above processes can thus play significant roles in shrinking it to the point where mass transfer begins. Recent numerical simulations suggest that tidal-capture binaries are born with orbital radii (or semi-major axes) in the range $1 < a_i/R_\odot < 15$ for main-sequence (henceforth ms) or early subgiant companions, and in the range $40 < a_i/R_\odot < 100$ for horizontal-branch companions (Portegies Zwart et.al. 1997). For binaries formed by exchange encounters, the orbital radii are generally expected to be somewhat larger than those for corresponding tidal binaries with identical members. Thus, both collisional hardening and gravitational radiation are expected to play major roles in the orbital shrinkage to Roche-lobe contact for most of the PXBs in GCs, whether dynamically formed or primordial.

Although we have included magnetic braking as above for completeness, its role in hardening of PXBs into XBs appears to be rather insignificant, at least for the VZ scaling adopted above. This is evident from the fact that there is little change in any of the results described here whether mag-

netic braking is included or not. For the VZ scaling, this is easy to understand. With a steep increase at small a , this effect is significant only at very small orbital separation, when the PXB has already come into Roche-lobe contact and become an XB. Thus, this process may well be significant in the further orbital evolution of the XB as mass-transfer proceeds (van den Heuvel 1991, van den Heuvel 1992), but not in the PXB-hardening process under study here.

Actually, further study of magnetic braking since the original VZ formulation has revealed many interesting points. The nature and strength of this effect very likely depends on the mass and evolutionary state of the companion star. For example, magnetic braking may become totally ineffective for very low-mass companions with $m_c \sim 0.3M_\odot$ or less, as these stars are fully convective. Whereas a significant convective envelope is necessary for strong magnetic braking, “anchoring” of the magnetic field in a radiative core is also believed to be essential for it, and it is argued that the effect would basically vanish when the star becomes fully convective (Spruit & Ritter 1983, Podsiadlowski et.al. 2002). Indeed, this forms the basis for the standard explanation for the *period gap* in CVs (van den Heuvel 1992 and references therein). Further, studies of the rotation periods of stars in open clusters have suggested that magnetic braking may be less effective than that given by the VZ prescription: this has been modelled in recent literature by either (a) the VZ scaling as above, but a smaller numerical constant than that given above, or, (b) a “saturation” effect below a critical value of a , wherein the scaling changes from the $VZ \sim a^{-4}$ scaling of Eq. (2) to a much slower $\sim a^{-1}$ scaling below this critical a -value (van der Sluys et.al. 2005). While these modifications are of relevance to XB evolution, it does not appear that they can alter the PXB-hardening results described here in any significant way. Accordingly, we shall not discuss magnetic braking any further here, and keep this term in the complete equations only to remind ourselves that it is operational, in principle, for companions with $m_c \geq 0.3M_\odot$.

2.1 An Interesting Scaling

Interplay between collisional hardening and gravitational-radiation hardening near the above critical orbital separation a_c produces a characteristic scaling, which we now describe. Consider the shrinkage time τ_{PXB} of a PXB from an initial orbital separation a_i to the final separation a_f corresponding to Roche-lobe contact and the onset of mass transfer, given by:

$$\tau_{\text{PXB}}(a_i, \gamma) \equiv \int_{a_i}^{a_f} \frac{da}{\dot{a}_{\text{GW}} + \dot{a}_{\text{MB}} + \dot{a}_{\text{C}}} \quad (7)$$

For given values of stellar masses, τ_{PXB} scales with the GC parameter γ introduced above as

$$\tau_{\text{PXB}} \sim \gamma^{-4/5}. \quad (8)$$

The scaling is almost exact at high values of γ , *i.e.*, $\gamma > 10^4$, say, there being a slight fall-off from this scaling at low γ 's.

How does this scaling arise? To see this, consider first the qualitative features of the integrand on the right-hand side of Eq. (7), *i.e.*, the reciprocal of the total shrinkage rate at an orbital separation a , which we denote by $\zeta(a)$, and

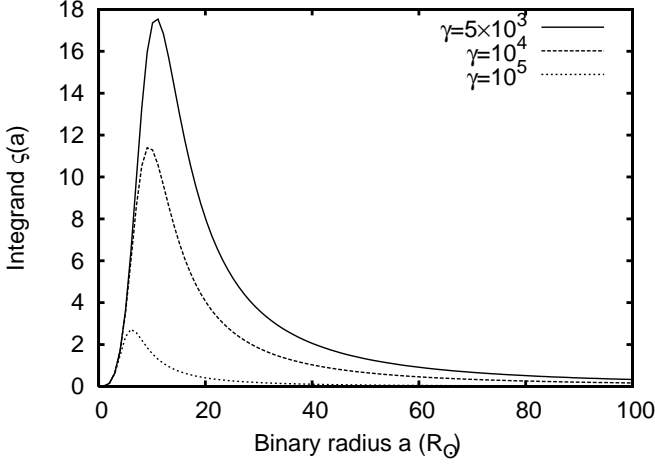


Figure 2. *Integrand $\zeta(a)$ in Eq. (7) shown as function of orbital separation a , with values of γ as indicated.*

which is displayed in Fig. 2. It is sharply peaked at $a \sim a_c$: indeed, the peak would be *exactly* at the above critical separation a_c but for the effects of magnetic braking, as can be readily verified. Since the latter effects are not important in the range of a -values relevant for this problem, as explained above, we can get a good estimate of the actual result by considering only gravitational radiation and collisional hardening. Because of this dominant, sharp peak in $\zeta(a)$, most of the contribution to the integral, *i.e.*, to τ_{PXB} , comes from there, provided that the integration limits (a_i, a_f) are such that *all or most of the peak is included*. We assume for the moment that this is so, and return to a discussion in the next subsection of what happens when this condition fails.

Under the above circumstances, we can immediately give a rough estimate of τ_{PXB} , which is the area under the curve in Fig. 2, as $\tau_{PXB} \sim 2a_c \times (\text{maximum value of the above integrand})$. This maximum value is simply $1/(2\alpha_{GW}a^{-3})$ if we neglect magnetic braking, since the gravitational radiation term equals the collisional hardening term there, as explained above. This gives $\tau_{PXB} \sim \alpha_{GW}^{-1}a_c^4$, which, with the aid of Eq. (6), yields $\tau_{PXB} \sim \alpha_{GW}^{-1/5}\alpha_C^{-4/5}\gamma^{-4/5}$. This is the basic reason for the scaling given by Eq. (8).

An exact evaluation of the integral in Eq. (7), with the magnetic braking term neglected, confirms this, as expected and as detailed in Appendix A. The exact result is:

$$\tau_{PXB} = \alpha_{GW}^{-1/5} \alpha_C^{-4/5} \gamma^{-4/5} [I(b_f) - I(b_i)]. \quad (9)$$

Here, b is a dimensionless orbital separation defined by $b \equiv a/a_c$, and the integral $I(x)$ is given in Appendix A. As $I(x)$ has only a logarithmic dependence on x under these circumstances, the basic scaling is $\tau_{PXB} \sim \gamma^{-4/5}$, as above. It is this basic scaling that leads to the essential behavior of the shrinkage time τ_{PXB} discussed in this paper.

2.2 Breakdown of Scaling?

When would the above scaling break down, and why? A simple answer is clear from Fig. 2: this would happen when the integration limits (a_i, a_f) are such that all or most of the above peak in $\zeta(a)$ is *not* included. For the present problem,

this basically reduces to an upper bound on a_f , since a_i is normally large enough to ensure that the region of integration in Fig. 2 extends well into considerably larger values of a beyond the peak. When a_f becomes so large as to exceed a_c , the region of integration is severely curtailed from the left in Fig. 2, so that most of the peak's contribution is missed, and the above scaling breaks down. We might think that such a situation would arise when the low-mass companion in the PXB is an evolved, horizontal-branch star, which has a much larger radius than a ms/subgiant companion of the same mass, and so would be expected to come into Roche-lobe contact at a much larger value of R_L , say $5 - 10R_\odot$, and correspondingly larger values of a_f . But such binaries are not relevant to our discussion here, since the lifetimes ($\approx 10^7$ y) of such horizontal-branch stars are too short to be of significance to the long binary-hardening timescales under consideration here. Thus, this possibility is not of practical importance here.

However, there *is* a situation in which this scaling is not relevant, not because it breaks down, but, rather because we move into a region of γ -values where τ_{PXB} computed in the above way exceeds the expected main-sequence lifetime τ_c of the low-mass ms/early-subgiant companion. Under these circumstances, the companion starts evolving into a giant and rapidly fills its Roche lobe, for essentially any value that a is likely to have at that stage. This is formally equivalent to saying that τ_{PXB} saturates at a value τ_c in this range of γ . We return to this point below.

2.3 Shrinkage Time

We now calculate the exact variation of the shrinkage time τ_{PXB} with the encounter-rate parameter γ introduced earlier, keeping all terms in Eq. (7). For this, we need to specify the initial and final values, a_i and a_f , of the orbital separation. We adopt $a_f \approx 1.94R_\odot$ for ms/subgiant companions corresponding to Roche-lobe contact, when the radius of the Roche-lobe R_L of the companion becomes equal to the radius of the companion itself. This translates into the above value of the orbital separation a_f by the well-known Paczyński relation:

$$R_L = 0.46a \left(\frac{m_c}{M} \right)^{1/3} \quad (10)$$

corresponding to R_L being equal to companion radius $\approx 0.6R_\odot$ for a companion mass $\approx 0.6M_\odot$.

In general, a_i will have a value which is within a possible range (a_i^{\min}, a_i^{\max}), which is indicated in Table 1. This range depends on the formation-mode of the binary, and also on the evolutionary status of the companion. The former has two possibilities, namely, (a) the binary is primordial, *i.e.*, it was already a binary when the globular cluster formed, or, (b) it formed by tidal capture or exchange interactions in the dense core of the globular cluster. The latter has also two basic possibilities, namely that the companion is either (a) a ms/subgiant, or, (b) a horizontal-branch star, as explained earlier. As explained above, however, the short lifetimes of horizontal-branch (Kippenhan & Weigert 1990, Clayton 1968) stars compared to the timescales of hardening processes under study here make it clear that they are of little importance in this problem, and we shall not consider them any further in this work. The ranges of a_i adopted in

Table 1. Distribution functions $f(a_i)$ and range of initial orbital separations a_i of compact-star binaries in globular clusters

Type of compact binary	Range of initial radius a_i	Form of distribution function $f(a_i)$
Dynamically formed compact star binary with ms or subgiant companion	$2R_\odot - 50R_\odot$	$f(a_i) \sim \frac{1}{a_i}$, $f(a_i) = \text{constant}$, $f(a_i) \sim a_i$, Gaussian in a_i with $\mu = 6.0R_\odot$ and $\sigma = 12.7R_\odot$
Primordial compact binaries	$2R_\odot - 500R_\odot$	$f(a_i) \sim \frac{1}{a_i}$

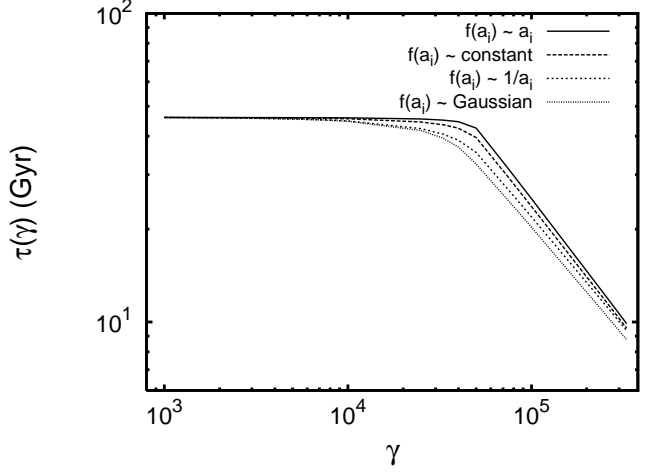
various cases are detailed in Table 1, and are taken from current literature (Portegies Zwart et.al. 1997).

Since a GC has a distribution of a_i s, we wish to study how $\tau_{PXB}(\gamma, a_i)$ averaged over such a distribution scales with γ , since both of these represent overall properties of the cluster. To this end, we define a suitable average shrinkage time as:

$$\tau(\gamma) \equiv \langle \tau_{PXB} \rangle \equiv \int_{a_i^{min}}^{a_i^{max}} \tau_{PXB}(\gamma, a_i) f(a_i) da_i, \quad (11)$$

where $f(a_i)$ is the normalized distribution of a_i in the range (a_i^{min}, a_i^{max}) . For this distribution, some indications and constraints are available, as follows. For primordial binaries, the distribution $f(a) \propto 1/a$ (corresponding to a flat cumulative distribution in $\ln a$) is well-established (Kraicheva et.al. 1978). For tidal capture/exchange binaries, results from numerical simulations like those of Portegies Zwart et.al. (1997) generally suggest a bell-shaped distribution over the relevant ranges for both ms/subgiant companions and horizontal-branch companions, although other distributions are not ruled out. To explore a plausible range of possibilities, we have studied the following distributions, as detailed in Table 1: (a) the above reciprocal distribution $f(a) \propto 1/a$, (b) a uniform distribution $f(a) = \text{const}$, (c) a linear distribution $f(a) \propto a$, and (d) a Gaussian distribution $f(a) \propto \exp[-(a - a_0)^2/\sigma^2]$ with appropriately chosen central value a_0 and spread σ given in Table 1. The ultimate purpose is to assess the sensitivity (or lack thereof) of the final results on this distribution, as we shall see.

Calculation of $\tau(\gamma)$ clarifies the following points. Primordial binaries have a range of a_i s whose upper limit is considerably larger than that for ms/early-subgiant binaries, but most of those binaries which lie between these two upper limits are too wide to be of any practical importance in this problem. Thus, it appears that we need consider in detail only PXBs with ms/early-subgiant companions for our purposes here, and Fig. 3 shows the distribution-averaged shrinkage time $\tau(\gamma)$ as a function of the encounter-rate measure γ for such binaries. As can be seen, the above $\gamma^{-4/5}$ -scaling is almost exact at high values of γ , say for $\gamma > 10^4$, there being a fall-off from this scaling at intermediate γ s, the extent of which depends on the case, as shown. We find that

**Figure 3.** $\tau(\gamma)$ vs. γ for PXBs: see text. Curves so normalized as to have the same “saturation value” $\tau_c = 45$ Gyr at low values of γ .**Table 2.** Values of γ_0 obtained by fitting equation (12) to computed $\tau(\gamma)$ vs. γ curves in Fig. 3

Type of initial distribution function	Value of γ_0
$f(a_i) \sim a_i$	8.49×10^3
$f(a_i) \sim \text{constant}$	8.74×10^3
$f(a_i) \sim 1/a_i$	1.21×10^4
$f(a_i) \sim \text{Gaussian}$	1.06×10^4

the above behavior can be well-represented by the analytic approximation

$$\tau(\gamma) \approx \frac{A_0}{\gamma_0^{4/5} + \gamma^{4/5}}, \quad (12)$$

where A_0 is a constant which depends on the range (a_i^{min}, a_i^{max}) (and also on the stellar masses, as explained above), and γ_0 depends on the above and also on the distribution $f(a_i)$. To illustrate the latter effect, we have given in Table 2 the inferred values of γ_0 , where the curves begin to deviate from the asymptote, for various distributions in the case of tidal capture/exchange binaries.

We can see the trend that, as the distribution of a_i tends to emphasize larger and larger values of a_i over the permissible range (as happens in going from a $f(a_i) \sim a_i^{-1}$ to a uniform distribution $f(a_i) \sim \text{const.}$, and further to a linear distribution $f(a_i) \sim a_i$), γ_0 decreases. The physical reason for this is straightforward. Collisional hardening, whose rate scales with γ , is dominant at large a (scaling as a^2 , as shown by Eqn. (3)). Hence, larger values of a_i increase the relative contribution of collisional hardening to τ , making it dominant over a larger range of γ , so that the asymptote $\tau \approx A_0 \gamma^{-4/5}$ corresponding to pure collisional hardening is followed over a larger range of γ , and so γ_0 becomes smaller. It follows that those distributions which emphasize larger values of a_i will lead to smaller values of γ_0 .

Finally, at low values of γ (about $10^3 - 3 \times 10^4$), the following aspect of the low-mass companion's evolutionary characteristics enters the picture. The value of $\tau(\gamma)$ calculated in the above manner then exceeds the main-sequence lifetime τ_c of the companion, a simple, widely-used estimate for which is

$$\tau_c \approx 13 \times 10^9 (m_c/M_\odot)^{-2.5} \text{ yr.} \quad (13)$$

For a typical low-mass companion with $M_c \approx 0.6M_\odot$, therefore, $\tau_c \approx 45$ Gyr. When $\tau(\gamma)$ calculated as above exceeds this value of τ_c , what happens is that the companion evolves into a giant, and so comes into Roche-lobe contact at a time $\approx \tau_c$ for essentially all plausible values of a at this point, irrespective of the calculated value of $\tau(\gamma)$. This is formally equivalent to the statement that $\tau(\gamma)$ reaches a saturation value of τ_c at low values of γ , the change-over occurring at $\gamma = \gamma_c$ such that $\tau(\gamma_c) = \tau_c$. Thus, the computed values can be analytically approximated by the prescription that $\tau(\gamma)$ is given by Eq. (12) for $\gamma > \gamma_c$, and by $\tau(\gamma) = \tau_c$ for $\gamma < \gamma_c$. This is shown in Fig. 3, where we normalize all the curves to the above, common “saturation value” $\tau_c = 45$ Gyr. Note that the lifetimes of GCs are typically $\sim 10 - 14$ Gyrs, so that, in a given GC, only those PXBs which reach Roche-lobe contact within its lifetime would be relevant for our purposes. What we have shown in Fig. 3 is the *formal* behavior of the distribution-averaged $\tau(\gamma)$ for plausible distributions of a_i . For a given GC, only that range of values of a_i which corresponds to Roche-lobe contact within its lifetime will go into the specific calculation for it.

3 EVOLUTION OF COMPACT-STAR BINARIES IN GLOBULAR CLUSTERS

How can we test the above scaling? Since τ is not directly observable, are there possible signatures that its scaling with γ might leave in the observed behavior of the populations of compact X-ray binaries in globular clusters? We briefly consider this question now and suggest possible answers.

As remarked earlier, PXBs are formed in GCs primarily by tidal capture and exchange interactions in the GC core. The rate of the former process is proportional to the encounter rate between single stars in the GC core. The latter rate is commonly denoted by Γ in the literature, and it scales as $\Gamma \propto \rho_c^2 r_c^3 / v_c$ with the average core density ρ_c , the velocity dispersion v_c of the stars in the core, and the core radius r_c . We can describe this as a rate of increase of the number n_{PXB} of PXBs in the GC which is $\alpha_1 \Gamma$, where α_1 is a constant. In an exchange interaction, one of the members of a binary consisting of two normal stars is replaced by a heavier compact star. The rate of this process is proportional to the encounter rate between the above two populations. Assuming the population of compact stars in a GC to scale with the entire stellar population in the GC, and also the population of normal-star binaries in a GC to scale with its total population, both of which are normally done, the rate of the exchange process also scales with the square of the stellar density, and therefore with the above Γ parameter, and we can express it in a similar vein as $\alpha_2 \Gamma$, where α_2 is another constant. Thus, we can write the entire formation rate phenomenologically as $\alpha \Gamma$, where $\alpha \equiv \alpha_1 + \alpha_2$.

After formation, two main processes affect the fate of

the PXBs. The first is the process of the hardening of the PXB to the point of Roche-lobe overflow and conversion into an XB, as discussed in this paper. This proceeds on a timescale τ_{PXB} , which means that the PXB population decreases on a timescale τ_{PXB} , which we can describe phenomenologically by a rate of decrease of n_{PXB} which is n_{PXB}/τ_{PXB} . This process may be slightly modified by second-order ones, which are normally ignored. For example, during the above hardening, an exchange encounter of the PXB with a single normal GC star heavier than the companion in the PXB can replace the latter.

The second process is the destruction of a PXB by its encounter with single stars in the GC core. This can happen in the two following ways. First, a fraction of the star-PXB encounters leads to a disruption or *ionization* of the PXB. This leads to a reduction in n_{PXB} , whose rate is proportional to γ , the star-binary encounter rate introduced earlier, and also to n_{PXB} . We can thus express this rate of decrease in n_{PXB} phenomenologically as $\beta_1 \gamma n_{PXB}$, where β_1 is a constant. Secondly, in a smaller fraction of such encounters, a compact star can replace the normal low-mass companion in an exchange encounter, resulting in the formation of a double compact-star binary. This is equivalent to destroying the PXB, since such a binary will not evolve into an XB. The rate of this process scales with both n_{PXB} and the rate of encounter between a given PXB and compact stars. If we again argue that the population of compact stars in a GC scales with the entire stellar population in it, this rate is $\propto \gamma$, and the rate of reduction of n_{PXB} can be written phenomenologically as $\beta_2 \gamma n_{PXB}$, where β_2 is a constant. The total PXB destruction rate can thus be written as $\beta \gamma n_{PXB}$, with $\beta \equiv \beta_1 + \beta_2$.

3.1 A Simple “Toy” Evolutionary Scheme

We now combine the above points into a simple “toy” description of PXB and XB evolution in GCs, which we can use in an attempt to extract possible signatures of the scaling described in this work. In this “toy” scheme, which is similar in spirit to that of White & Ghosh (1998) and Ghosh & White (2001) for following the evolution of X-ray binary populations of galaxies outside GCs, the evolution of the PXB population is given by:

$$\frac{\partial n_{PXB}}{\partial t} = \alpha \Gamma - \beta \gamma n_{PXB} - \frac{n_{PXB}}{\tau_{PXB}}. \quad (14)$$

wherein the above rates of increase and decrease of n_{PXB} have simply been collected together.

The evolution of the XB population n_{XB} resulting from the above PXBs is described in a similar manner:

$$\frac{\partial n_{XB}}{\partial t} = \frac{n_{PXB}}{\tau_{PXB}} - \frac{n_{XB}}{\tau_{XB}}. \quad (15)$$

Here, τ_{XB} is the evolutionary timescale for XBs. The idea here is that XBs are created from PXBs at the rate n_{PXB}/τ_{PXB} , and conclude their mass-transfer phase, and so their lifetime as XBs, on a timescale τ_{XB} .

In the spirit of the “toy” model, all timescales in equations (14) and (15) can be considered constants, as can α and β , while in reality they depend on orbital parameters and stellar properties, as also on other parameters. These equations can then be solved readily, and of interest to us here

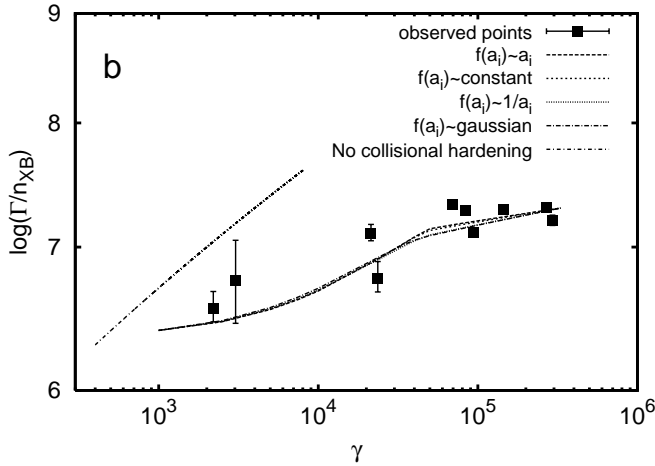


Figure 4. Γ/n_{XB} vs. γ for galactic globular clusters. Observational points with error bars from Pooley et.al. (2003). Trend suggested by Eq. (17) shown for various distributions as indicated. Also shown is the trend expected in absence of collisional hardening.

is the asymptotic behavior, obtained by setting the time-derivatives to zero in these, which yields an XB population:

$$n_{XB} = \frac{\alpha \Gamma \tau_{XB}}{1 + \beta \gamma \tau(\gamma)}. \quad (16)$$

The effect of collisional hardening is immediately seen on the right-hand side of Eqn. (16), in the second term in the denominator. Since collisional hardening always decreases τ , it increases n_{XB} , other things being equal. This enhancement in n_{XB} is as expected, as collisional hardening makes a larger number PXBs reach Roche-lobe contact. Thus, in a GC with given properties, the number of X-ray sources is expected to be enhanced by collisional hardening compared to what it would be if this effect were negligible.

3.2 Signature of Collisional Hardening?

Can we look for observational evidence of the above enhancement in XB populations of GCs expected from collisional hardening? We discuss briefly an attempt to use *Chandra* observations of GCs to this end, with the cautionary remark that our evolutionary model, as given in the previous subsection, is still too simple-minded to apply quantitatively to actual GC data. What we are looking for, therefore, is a possible *qualitative* trend that is consistent with the ideas of collisional hardening discussed in this paper, which will encourage us to perform a more detailed study.

The trend given by Eqn. (15) readily translates into one of the form

$$\frac{\Gamma}{n_{XB}} = A + B\gamma\tau(\gamma), \quad (17)$$

where $A \equiv 1/\alpha\tau_{XB}$ and $B \equiv \beta A$ are constants, independent of the cluster parameters. We can compare this with data obtained from *Chandra* observations of GCs, as given in Pooley et.al. (2003). This is shown in Fig. 4.

It is clear that the trend suggested by Eq. (17) is consistent with the data, while that expected when collisional hardening is entirely neglected is not. The flattening of the

trend on inclusion of collisional hardening is precisely due to the scaling discussed in this paper. However, we stress again that ours is only a “toy” model at this stage, relevant only for exploring feasibility. To study the effect of a_i distributions, we have normalized the constants A and B for each distribution by having the curve pass through two chosen points at the lowest and highest values of γ for which data is available. The results show clearly that varying the distribution has almost no effect on the trend.

4 DISCUSSIONS

The purpose of this paper is to point out an essential effect in the hardening of PXBs in GCs, *viz.*, that collisional hardening *increases* with increasing a and orbital period, while that due to gravitational radiation has the opposite trend, so that their interplay leads to a characteristic scaling of the total hardening rate with GC parameters. In our introductory treatment of this effect here, we have given very simple formulations of many physical processes. First, collisional hardening is an inherently stochastic process, wherein individual events of varying sizes accumulate to yield a final state, and the Shull (1979) rate we have used is a continuous approximation to it. Secondly, the essential two- and three-body interactions that determine the evolution and fate of a PXB in a GC are also stochastic by nature. For example, approximating an ionization event — in which a binary is disrupted — by a continuous term is necessarily a great oversimplification. Thus, an improved treatment must include a proper formulation of these stochastic processes, which we are developing currently.

Thirdly, we have confined ourselves to circular orbits here, while binaries created by tidal capture and/or exchange interactions often have quite eccentric orbits, in which tidal circularization must play a dominant role during initial phases of hardening. Fourthly, mass segregation is an essential effect in GCs, which reflects itself in the accumulation of the heaviest objects in the core of a GC, and so in a change in the effective mass-function in the core. Finally, the evolution of the GC must be taken into account in a realistic calculation: this would make the GC parameters time-dependent, while we have treated them as constants here, and may indeed have a large effect if core collapse and bounce are modelled.

The above processes will need to be adequately modelled in a realistic calculation of the evolution of PXBs and XBs in GCs. Such calculations are under way, and the results will be reported elsewhere.

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REFERENCES

- Angelini L., Loewenstein M. and Mushotzky R.F., 2001, *ApJ*, 557, L35.
- Ashman K.M. and Zepf S.E., 1998, *Globular Cluster Systems*, Cambridge Univ. Press.
- Clayton D., 1968, *Principles of Stellar Evolution and Nucleosynthesis*, McGraw-Hill, New York.

- d’Inverno R., 1992, “Introducing Einstein’s Relativity”, Oxford Univ. Press.
- Ghosh P. and White N.E., 2001, ApJ, 559, L97.
- Goodman J. and Hut P., 1993, ApJ, 403, 271.
- Gradshteyn I.S. and Ryzhik I. M., 1980, Table of Integral, Series, and Products, Academic Press, San Diego.
- Heggie D., 1975, MNRAS, 173, 729.
- Hut P., 1985, IAUS, 113, 231H.
- Hut P. and Verbunt F., 1983, Nature, 301, 587.
- Hut P., McMillan S. and Romani R.W., 1992, ApJ, 389, 527.
- Hut P. et. al., 1992, PASP, 104, 981H.
- Kippenhan R. and Weigert A., 1990, Stellar Structure and Evolution, Springer-Verlag, Berlin.
- Kraicheva Z.T., Popova E.I., Tutukov A.V. and Yungelson L.R., 1978, in “Nonstationary Evolution of Close Binaries”, ed. Zytkov A., Polish Academy of Sciences, p.25
- Pooley D. et.al., 2003, ApJ, 591, L131.
- Podsiadlowski Ph., Rappaport, S. and Pfahl, E. D., 2002, ApJ, 565, 1107.
- Portegies Zwart S.F., Hut P., McMillan S.L.W. and Verbunt F., 1997, A&A, 328, 143.
- Portegies Zwart S.F. and Meinen A.T., 1993, A&A, 280, 174.
- Sarazin C.L. et.al., 2003, ApJ, 595, 743.
- Shull J.M., 1979, ApJ, 231, 534.
- Spitzer L. Jr., 1987, Dynamical Evolution of Globular Clusters, Princeton Univ. Press.
- Spruit, H.C. and Ritter, H., 1983, A&A, 124, 267.
- van den Heuvel E.P.J., 1991, in “Neutron Stars: Theory and Observation”, eds. Ventura J. and Pines D., Kluwer, Dordrecht, p.171.
- van den Heuvel E.P.J., 1992, in “X-Ray Binaries and Recycled Pulsars”, eds. van den Heuvel E.P.J. and Rappaport S.A., Kluwer, Dordrecht, p.233.
- van der Sluys M. V., Verbunt F. and Pols, O.R. 2005, A&A, 440, 973.
- Verbunt F., 2002, New horizons in globular cluster astronomy, ASP conf. series.
- Verbunt F. and Zwaan C., 1981, A&A, 100, L7.
- Verbunt F. and Hut P., 1987, IAUS., 125, 187V.
- Verbunt F. and Lewin W.H.G., 2004, in “Compact Stellar X-ray Sources”, eds. Lewin W.H.G. and van der Klis M., Cambridge Univ.Press.
- White N.E. and Ghosh P., 1998, ApJ, 504, L31.
- White R.E., Sarazin C.L. and Kulkarni S.R., 2002, ApJ, 571, L23.

Defining $\delta \equiv B\gamma$ and substituting $\delta a^5 \equiv b^5$ in the above, we get

$$I_1 = \delta^{-\frac{4}{5}} [I]_{b_f}^{b_i}, \quad (\text{A3})$$

where the indefinite integral $I(x)$ is given by,

$$I = \int \frac{x^3 dx}{1+x^5} \quad (\text{A4})$$

Standard expressions for integrals of type I are given in Gradshteyn and Ryzhik (1980), from which we get,

$$I(x) = -\frac{1}{5} \ln(1+x) - \frac{1}{5} \left[\cos \frac{\pi}{5} \ln \left(1 - 2x \cos \frac{\pi}{5} + x^2 \right) + \cos \frac{2\pi}{5} \ln \left(1 + 2x \cos \frac{2\pi}{5} + x^2 \right) \right] \quad (\text{A5})$$

$$+ \frac{2}{5} \left[\sin \frac{\pi}{5} \tan^{-1} \left(\frac{x - \cos \frac{\pi}{5}}{\sin \frac{\pi}{5}} \right) + \sin \frac{2\pi}{5} \tan^{-1} \left(\frac{x + \cos \frac{2\pi}{5}}{\sin \frac{2\pi}{5}} \right) \right]$$

From equations (A1), (A2), (A3), (A5),

$$\tau_{PXB}(a_i, \gamma) = \alpha_C^{-\frac{4}{5}} \alpha_{\text{GW}}^{-\frac{1}{5}} \gamma^{-\frac{4}{5}} [I]_{x=b_f}^{x=b_i} \quad (\text{A6})$$

APPENDIX A: ANALYTICAL EXPRESSION FOR $\tau(\gamma)$

We drop the magnetic braking term in the integral on right-hand side of Eqn. (7), as explained in the text, and obtain:

$$\tau_{PXB}(a_i, \gamma) \approx \int_{a_f}^{a_i} \frac{da}{\alpha_{\text{GW}} a^{-3} + \alpha_C a^2 \gamma} = \frac{1}{\alpha_{\text{GW}}} I_1, \quad (\text{A1})$$

where,

$$I_1 \equiv \int_{a_f}^{a_i} \frac{da}{a^{-3} + B a^2 \gamma}, \quad B \equiv \frac{\alpha_C}{\alpha_{\text{GW}}} \quad (\text{A2})$$